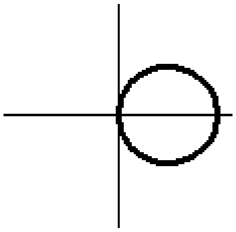
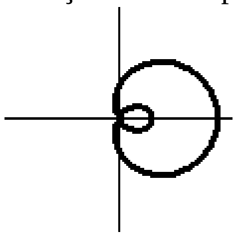
HOW TO GRAPH POLAR FUNCTIONS OF THE FORM $r = a \pm b\cos\theta$ OR $r = a \pm b\sin\theta$

1. Determine the shape of the graph by finding $\left|\frac{a}{b}\right|$.

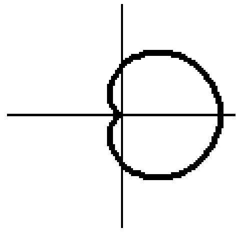
$\left|\frac{a}{b}\right| = 0$
circle



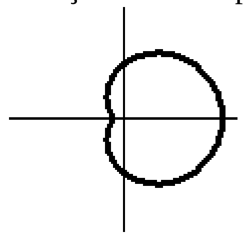
$\left|\frac{a}{b}\right| < 1$
limaçon with loop



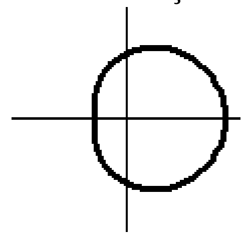
$\left|\frac{a}{b}\right| = 1$
cardioid



$1 < \left|\frac{a}{b}\right| < 2$
limaçon with dimple

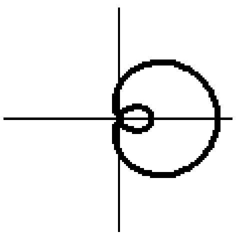


$\left|\frac{a}{b}\right| \geq 2$
convex limaçon

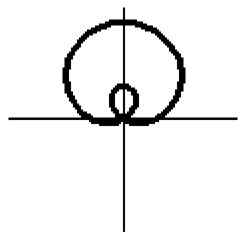


2. Determine the axis of symmetry by the trigonometric function used.

$r = a \pm b\cos\theta$
symmetric over polar axis ($x-$axis)

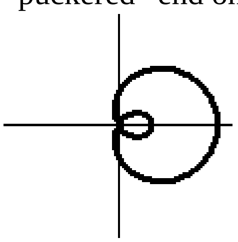


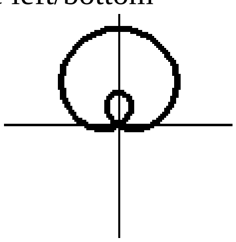
$r = a \pm b\sin\theta$
symmetric over $\theta = \frac{\pi}{2}$ ($y-$axis)



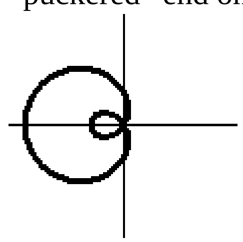
3. Determine the "direction" of the graph by the sign of b.

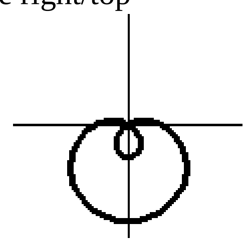
$b > 0$
"bigger" end on the right/top
"puckered" end on the left/bottom





$b < 0$
"bigger" end on the left/bottom
"puckered" end on the right/top





4. Determine the $x-$ and $y-$intercepts by finding the points corresponding to $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$\theta = 0, \pi$ correspond to positive and negative $x-$intercepts (assuming $r > 0$)
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ correspond to positive and negative $y-$intercepts (assuming $r > 0$)

If $r < 0$, the intercept is on the "other side" (negative vs positive, and vice versa) of the corresponding axis.

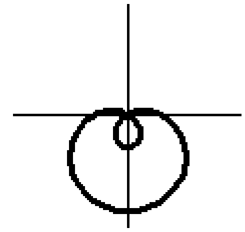
The origin is not an intercept for limaçons with dimples and convex limaçons,
but is an intercept for the other shapes.

Example: Graph $r = 2 - 3\sin\theta$

1. $\left|\frac{2}{-3}\right| < 1$ limaçon with loop

2. equation uses $\sin\theta$ symmetric over $\theta = \frac{\pi}{2}$ (y – axis)

3. $-3 < 0$ "bigger" end on the bottom, "puckered" end on the top

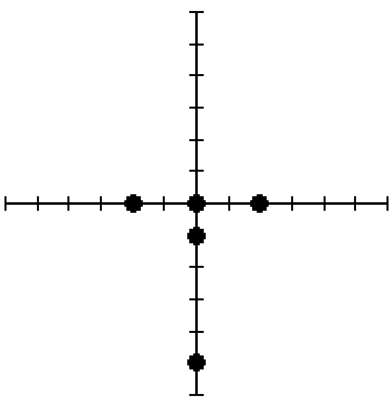


4.

θ	$r = 2 - 3\sin\theta$
0	2 (positive x – intercept)
$\frac{\pi}{2}$	–1 (negative y – intercept)
π	2 (negative x – intercept)
$\frac{3\pi}{2}$	5 (negative y – intercept)

origin is also an intercept (since shape is limaçon with loop)

Intercepts only



Complete graph